inversion technique was used to solve for the frequency responses.[2]

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# Predicted Coolant Heat Transfer and Friction Compared with Nuclear Rocket Test Results

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### Nomenclature

$A$ = cross-sectional area
$C_g$ = coefficient in heat-transfer equation
$c_1$ = exponent of $T_w/T_b$, Eq. (1)
$D$ = diameter
$e$ = relative roughness of surface
$F$ = entrance effect coefficient
$f$ = friction coefficient
$G$ = weight flow rate per unit cross-sectional area
$g$ = gravitational conversion factor
$Nu$ = Nusselt number
$Pr$ = Prandtl number
$\Delta p$ = total static pressure drop
$\Delta p_{fr}$ = friction pressure drop
$\Delta p_{mom}$ = momentum pressure drop
$R$ = radius of curvature
$Re$ = Reynolds number
$r$ = inside radius of passage
$S$ = surface area
$T$ = temperature
$w$ = weight flow rate
$x$ = linear distance from entrance
$\rho$ = density

#### *Subscripts*

$C,M$ = calculated and measured, respectively
$b,c$ = bulk and crown, respectively
$g,w$ = gas and wall, respectively
$s$ = straight tube or reference value

### Introduction

AN equation based on all available heat-transfer data for single-phase hydrogen flow through straight tubes was shown earlier[1] to correlate the data for a wide range of conditions including heat fluxes up to 27.6 Btu/sec/in.$^2$ (Ref. 2). The equation also was modified to include the effects of curvature[3] and compared with experimental data for single curved tubes.[2,4] The results and recommended applications of the equations have been reported.[5] Another equation[6] based on all available friction coefficient data for straight tubes was modified to include the effects of curvature.[3] In this Note these correlation equations are brought together and used to revise the coolant side calculations in an existing digital computer program for design and evaluation of convectively cooled rocket nozzles.[7] Gas-side wall temperature, coolant pressure drop, and coolant temperature rise predicted by this revised code are compared with measured values from several NERVA nuclear tests. Wall temperatures were not measured in Phoebus 2 and Pewee nuclear tests, so that only measured coolant temperature rise and pressure drop are compared with predictions. It should be noted that, although Pewee, NERVA, and Phoebus 2 vary a great deal in size and power, their nozzles have similar geometry; these calculation procedures should be compared with experiments for other geometries when available.

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### Basic Equations

#### *Coolant side*

Heat-transfer coefficients in the coolant passages are calculated using the straight-tube correlation equation[1]

$$Nu_{b_s} = 0.023 Re_b^{0.8} Pr_b^{0.4} (T_w/T_b)^{-c_1} \tag{1}$$

where $c_1 = 0.57 - 1.59\,D/x$, and modifying it[5]

$$Nu_b = Nu_{b_s}(1 + F_1 D/x) \tag{2}$$

where $F = 2.3$ for a 45° angle bend and 5 for a 90° angle bend.

The Itō correction for the effects of curvature on friction coefficients[3] has been applied to local heat-transfer coefficients with good results[5] using Eq. (1) to give

$$Nu_b = Nu_{b_s}[Re_b(r/R)^2]^n \tag{3}$$

where

$$n = 0.05 \text{ for the concave side (throat)} \tag{3a}$$

$$n = -0.05 \text{ for the convex side (knuckle)} \tag{3b}$$

Equations (1–3a) cover all the conditions encountered in the cooling passages of nuclear rocket nozzles.

Measured friction coefficients can be as much as three times the values predicted by conventional methods.[6] Reference 6 recommended

$$f_s/2 = (0.0007 + 0.0625/Re_w^{0.32})(T_w/T_b)^{-0.5} \tag{4}$$

For $Re_w \geq 3000$, Eq. (4) correlated all of the friction coefficients within $\pm 10\%$. It can be used at $x/D$ as low as 3 by using $c_1$ as the exponent of $T_w/T_b$. The Kármán-Nikuradse relation can also be modified with $(T_w/T_b)^{-c_1}$ to give

$$f_s = [4.0 \log(Re_w f_s^{1/2}) - 0.40]^{-2}(T_w/T_b)^{-c_1} \tag{5}$$

Assuming that the effect of $T_w/T_b$ is the same for a rough tube as it is for a smooth tube, the equation for a rough tube[7] becomes

$$f_s = [-4.0 \log(e/3.7D + 1.255/Re_w f_s^{1/2})]^{-2}(T_w/T_b)^{-c_1} \tag{6}$$

where $e$ is the relative roughness of the tube. The Itō correction for the effect of curvature[3] should be used with Eqs. (5) and (6) as follows:

$$f = f_s[Re_w(r/R)^2]^{0.05} \tag{7}$$

The exponent in Eq. (7) is 0.05 for both the throat and knuckle regions because, unlike the heat-transfer coefficient which is affected by only the heated surface, $f$ is affected by the wetted surface of the coolant passage. How well the Itō correction applies to local heat transfer and friction has not been studied thoroughly due to the paucity of local experimental data.

The friction and momentum pressure drops are calculated from

$$\Delta p_{fr} = 2fG^2x/Dg \tag{8}$$

$$\Delta p_{mom} = w^2/gA[(\rho A)_2^{-1} - (\rho A)_1^{-1}] \tag{9}$$

The total pressure drop is the sum, $\Delta p_{fr} + \Delta p_{mom}$.

***Hot-gas side***

In calculating the heat transfer from the hot gas to the cooled nozzle wall, it is common practice to use the Nusselt equation

$$Nu_f = C_g Re_f^{0.8} Pr_f^{0.4} \tag{10}$$

and vary the $C_g$ as a function of axial location or area ratio. The value of $C_g$ can either be measured[8] or calculated, e.g., Bartz boundary-layer calculations.[9] Because values for $C_g$ are usually adjusted after rocket tests to give calculated gas-side wall temperatures equal to measured values, $C_g$ thus becomes a function of coolant-side heat-transfer equations. The theoretical values of $C_g$ calculated by the Aerojet General Corp. for NERVA and Phoebus 2 nuclear runs are shown in Fig. 1; $C_g$ increases in the nozzle chamber from knuckle to reactor core face, but the area ratio remains constant, which is indicated by the vertical line in Fig. 1a. The area ratio of the nozzle chamber is not the same for NERVA, Phoebus 2, and Pewee.

For the NERVA nozzle, Fig. 1b, the adjusted $C_g$ is about 2.5 times the theoretical value in the knuckle region. The need for such a high $C_g$ in this region probably results from the lack of previous coolant-side correlation equations to take into account the decrease in heat transfer resulting from the convex curvature. The effect of curvature appears in Eq. (3). The $C_g$ values shown for the Pewee nozzle were experimentally determined by Los Alamos Scientific Laboratory. The data were reported by Schacht, Quentmeyer, and Jones.[8]

In Eq. (10), $Re_f$ is based on nozzle diameter. It would be more logical to treat the constant diameter chamber section as a flat plate and base $Re_f$ on axial distance along the wall were it not for the introduction of cold hydrogen for peripheral film cooling of that section.

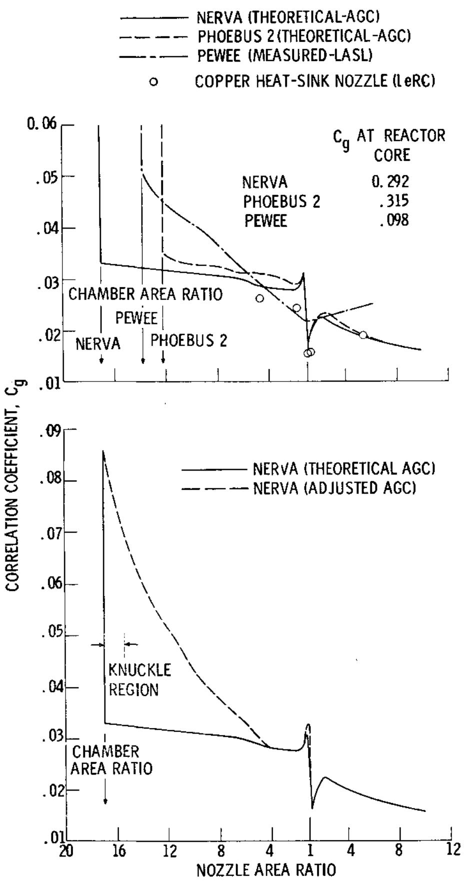
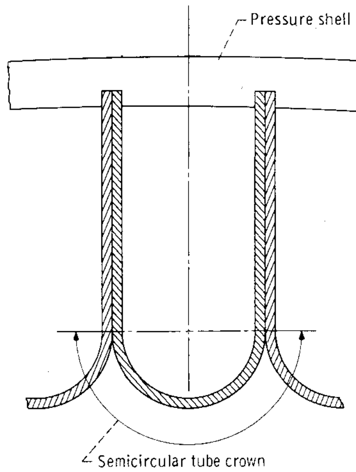



**Fig. 1 Variation of various correlation coefficients for hot-gas side heat transfer as a function of area ratio.**

**Fig. 2 Typical U-tube coolant passage of a nuclear rocket nozzle.**




## Nozzle Calculations

The nozzle calculations were made using a digital computer program[7] modified to incorporate the equations recommended in this paper. For the coolant side, a Nusselt type correlation equation with the physical properties evaluated at the film temperature is part of the computer program[7]; it was replaced by Eq. (1) with the effects of entrance configuration and curvature being applied in the proper regions as shown in Fig. 3. The friction coefficients were calculated using the Kármán-Nikuradse relation for smooth tubes and the rough tube equation which was modified to become Eq. (6), using values for the relative roughness $e$ of the coolant passages as follows: NERVA, 5 $\mu$in.; Phoebus 2, 10 $\mu$in.; and Pewee, 20 $\mu$in.

The surface temperature of the coolant passage varies as much as 1000°R from the crown (the semicircular portion of the U-tube) to the pressure shell (see Fig. 2). This temperature variation makes the calculation of a single $f$ for the complete wetted perimeter very difficult. Calculated $\Delta p$'s are

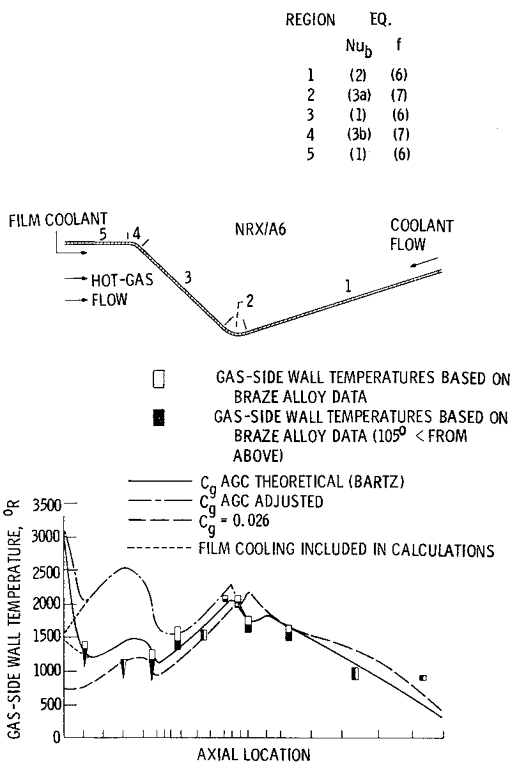



**Fig. 3 Comparison of calculated gas-side wall temperatures with measured values for NRX-A6 nuclear test, NERVA nozzle.**

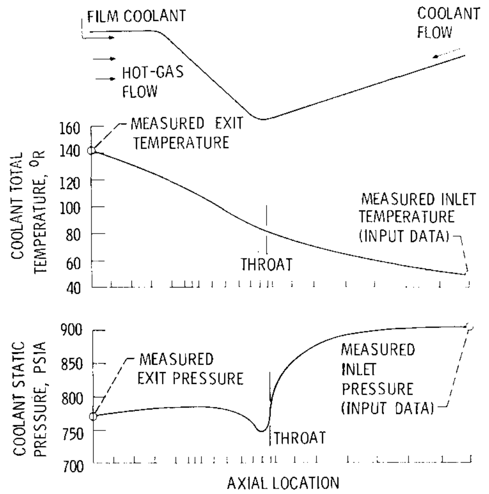



**Fig. 4 Variation of coolant temperature and pressure with axial location: NRX-A6 nuclear test, NERVA nozzle.**

often 60% lower than the measured values. The following results indicate the variations in $f$ that can result from two different assumptions for a section of coolant passage well downstream of the entrance region and upstream of the throat curvature in the NERVA nozzle using Eq. (6).

Assumption 1: $T_w$ is constant, and $T_w = T_b \therefore T_w/T_b = 1$; then $Re_w = Re_b = 2.7 \times 10^7$, and $f = 0.00267$.

Assumption 2: $T_w$ is constant and equals the crown temperature; thus, $T_w = 1200°\mathrm{R}$, $T_b = 80°\mathrm{R}$, $T_w/T_b = 15$, and $Re_w = 8.9 \times 10^4$, yielding $f = 0.0009$, or $\frac{1}{3}$ the value obtained by assumption 1.

The method used herein was to calculate one friction coefficient for the bulk temperature $f_b$ and another for the hot crown temperature $f_c$ for each station. Then an effective friction coefficient was defined as

$$f_{\mathrm{eff}} = f_b[S_b/(S_b + S_c)] + f_c[S_c/(S_b + S_c)] \qquad (11)$$

where $S_b$ and $S_c$ are the surface areas at bulk and crown temperatures, respectively (Fig. 2). Values of $S_c/(S_b + S_c)$ varied from 0.40 at the coolant entrance to 0.22 at the nozzle throat and 0.60 in the chamber of the NERVA nozzle. Equation (11) should be as applicable to a rectangular (or any other shape) coolant passage as to the U-tube passage.

The momentum pressure drop equation in the computer program[7] remained unchanged.

For the hot-gas side, the Nusselt type equation in the original program[7] was used, with various $C_g$'s (input data). The effect of peripheral film cooling in the chamber was calculated using the method of Hatch and Papell.[10] In the NERVA and Phoebus 2 nozzles the film coolant was assumed to affect $T_w$ for a distance of 6 in. from the reactor core. For Pewee, the film coolant was assumed to have some effect all the way to the knuckle because of higher coolant flow rate and lower coolant temperature. Gas static temperatures and pressures were calculated for hydrogen at equilibrium conditions for assigned chamber pressure, chamber temperature, and nozzle area ratios using the computer program of Ref. 11.

## Discussion of Results

The only experimental wall temperatures available from nuclear tests are for the NERVA nozzles NRX-A3, EST, A5, and A6. Gas-side wall temperatures predicted using the Aerojet General Corp. theoretical $C_g$, the AGC adjusted $C_g$, and a constant $C_g$ of 0.026 are compared to experimental gas-side wall temperatures for NRX-A6 in Fig. 3. The experimental wall temperatures resulted from an examination of braze alloy patches at various locations in the nozzle. The open and solid symbols are for different angular locations (105° apart) in the nozzle. The solid line represents the $T_w$'s calculated without film cooling using the theoretical $C_g$'s and is in very good agreement with the measured values for most of the length. The $T_w$'s calculated with and without film cooling using the adjusted values of $C_g$ were much higher than the measured values in the chamber and knuckle regions because of a) high values of $C_g$ in the adjusted curve (see Fig. 1b), and b) a decrease in the coolant heat-transfer coefficient resulting from the convex curvature effect in Eq. (3b). It appears that the effect of convex curvature has been included twice, once on the hot-gas side and once on the coolant side.

**Table 1 Calculated and measured temperature rise and pressure drop across coolant passages for nuclear tests**

| | Pressure drop | | | Temperature drop | | |
|---|---|---|---|---|---|---|
| Nozzle | Calculated $\Delta p_C$, psi | Measured $\Delta p_M$, psi | Error in $\Delta P_C$, % | Calculated $\Delta T_C$, °R | Measured $\Delta T_M$, °R | Error in $\Delta T_C$, % |
| NERVA | | | | | | |
| NRX-A3 | 137 | 134 | 2.2 | 87 | 89 | −2.2 |
| NRX-A4 | 153 | 179 | −14.5 | 90 | 88 | 2.3 |
| NRX-A5 | 144 | 141 | 2.1 | 94 | 103 | −8.7 |
| NRX-A6 | 139 | 141 | −1.4 | 95 | 93 | 2.1 |
| Phoebus-2, EP-IV | | | | | | |
| 1 | 27 | 29 | −6.9 | 26 | 13 to 36 | ... |
| 2 | 60 | 66 | −9.1 | 31 | 28 to 41 | ... |
| 3 | 132 | 151 | −12.6 | 55 | 38 to 62 | ... |
| 4 | 182 | 204 | −10.8 | 52 | 33 to 60 | ... |
| 5 | 211 | 255 | −17.3 | 78 | 57 to 88 | ... |
| 6 | 246 | 290 | −15.2 | 83 | 60 to 93 | ... |
| Pewee, EP-III | | | | | | |
| 37 | 59 | 62 | −4.8 | 62 | 56 | 10.7 |
| FP1A | 117 | 126 | −7.1 | 94 | 91 | 3.3 |
| FP1B | 108 | 118 | −8.5 | 93 | 92 | 1.1 |
| FP1C | 113 | 124 | −8.9 | 94 | 93 | 1.1 |
| FP1D | 113 | 124 | −8.9 | 94 | 93 | 1.1 |
| 341 | 31 | 37 | −16.2 | 23 | 24 | −4.2 |
| FP2A | 113 | 123 | −8.1 | 94 | 93 | 1.1 |
| FP2B | 113 | 125 | −9.6 | 94 | 96 | −2.1 |
| FP2C | 113 | 125 | −9.6 | 94 | 96 | −2.1 |
| FP2D | 113 | 125 | −9.6 | 94 | 97 | −3.1 |
| 342 | 31 | 37 | −16.2 | 23 | 25 | −8.0 |

It is very difficult to compare the predicted and measured wall temperatures near the core because the cold hydrogen used for peripheral film cooling is introduced at this point. Predicted wall temperatures are shown with and without peripheral film cooling. Experimental studies have shown that the inclusion of a simulated reactor core can increase the heat-transfer coefficients in a nozzle chamber.[12]

The dashed line in Fig. 3 represents the wall temperature calculated using a constant $C_g$ of 0.026. This simplified method represents the wall temperatures fairly well, except that the predicted values near the core were low, even though film cooling was not included in these calculations.

Calculated and measured pressure drops and temperature rises for the various test nozzles are shown in Table 1; calculated variations along the length of the nozzle are shown for the NRX-A6 in Fig. 4. The fact that calculated pressure drops are in almost all tests about 10% lower than the measured values indicates that the method of weighting the friction coefficients may overcorrect the effective coefficients in Eq. (11). Another possibility is that the effect of $T_w/T_b$, which has been verified for smooth tubes with temperature ratios up to 7.35, cannot be extrapolated to rough passages with temperature ratios of 15.

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# Hypersonic Aerodynamic Characteristics of Flat Delta and Caret Wing Models at High Incidence Angles

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RECENT proposals concerning the space shuttle vehicle have revived interest in the aerodynamics of efficient lifting re-entry configurations. The flat-bottom delta has been extensively studied as a promising shape for the wide range of flight conditions encountered by the shuttle. An experimental study by Opatowski[1] has indicated that, in cruising flight, the caret wing has some advantage over the flat delta. There is, however, a lack of information on the force characteristics of caret wings under conditions appropriate to re-entry.

As a consequence of growing three-dimensionality and flow spillage from the windward surface at high incidence angles, the lift on flat delta wings is known to drop increasingly below the two-dimensional shock value. It appears reasonable to expect that a caret wing designed to support a two-dimensional shock at a high incidence angle will develop a significantly higher lift coefficient than the flat delta.

Based on such assumptions, an analysis by Townend[2] leads to the conclusion that the caret wing (or its variants) may prove superior to the conventional delta wing in re-entry performance.

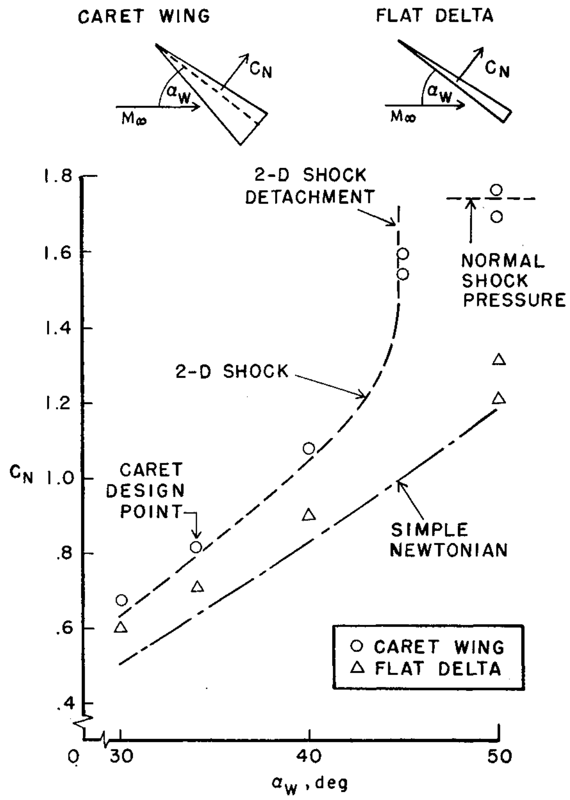


**Fig. 1 Normal force measurements.**

This Note presents the results of comparative balance measurements on flat and caret delta wing models (with sharp leading edges swept back at 75°) undertaken to verify the preceding assumption, and also to obtain some quantitative indications. The tests were carried out in the 8-in. Gun Tun-

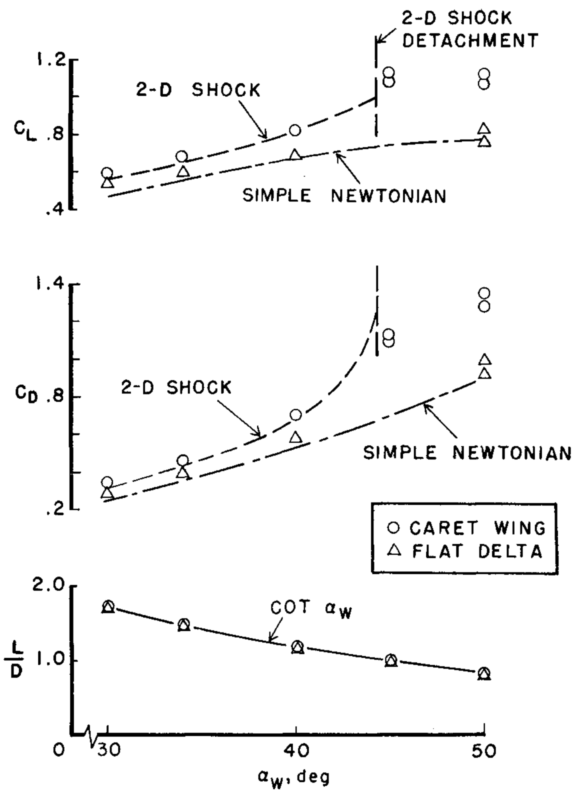


**Fig. 2 Lift and drag data.**

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